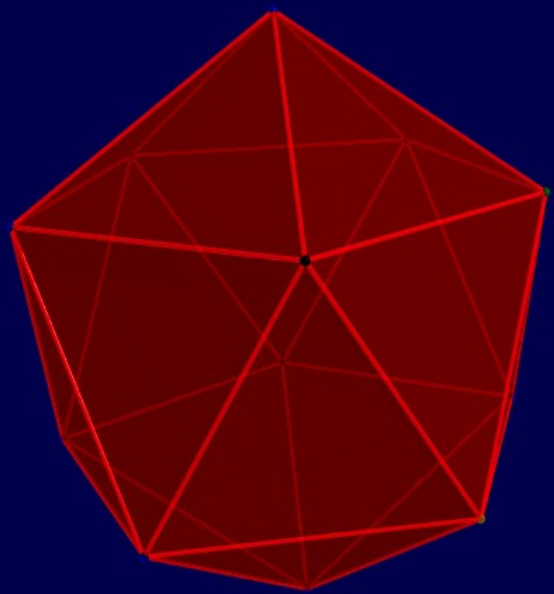
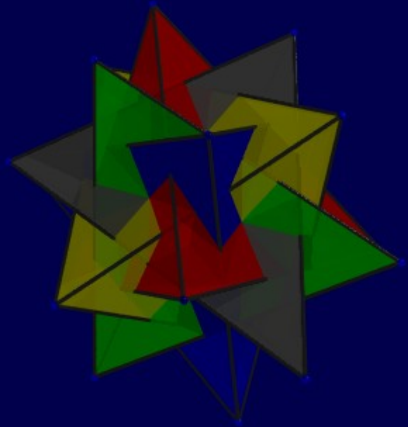
Icosahedron





Motivations:

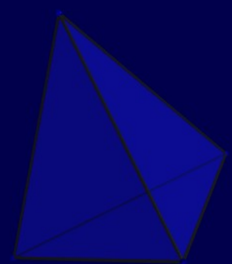
- Lobatschewsky: “There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world.”
- Plato’s cosmology
- Realization of a permutation group
- Poincare's theorem and conjecture

Definitions

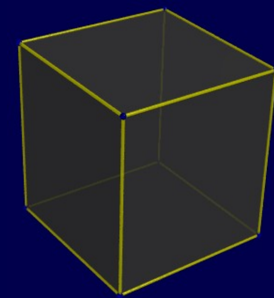
- Regular polygons: equiangular & equilateral
- Regular polyhedron: faces are congruent regular polygons which are assembled in the same way around each vertex
- Icosahedron:
  - 12 vertices, 30 edges, 20 faces
  - Face: equilateral triangles

http://en.wikipedia.org/wiki/Icosahedron

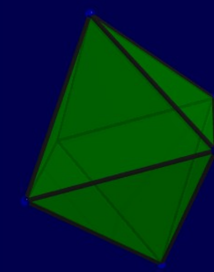
5 regular polyhedrons:



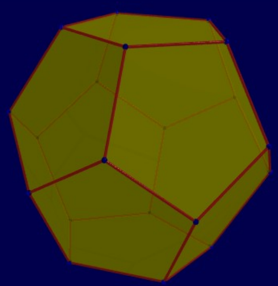
Tetrahedron



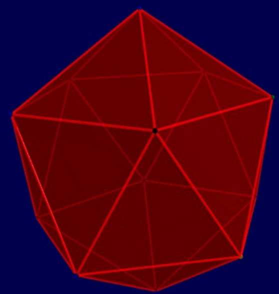
Cube



Octahedron



Dodecahedron



Icosahedro
n

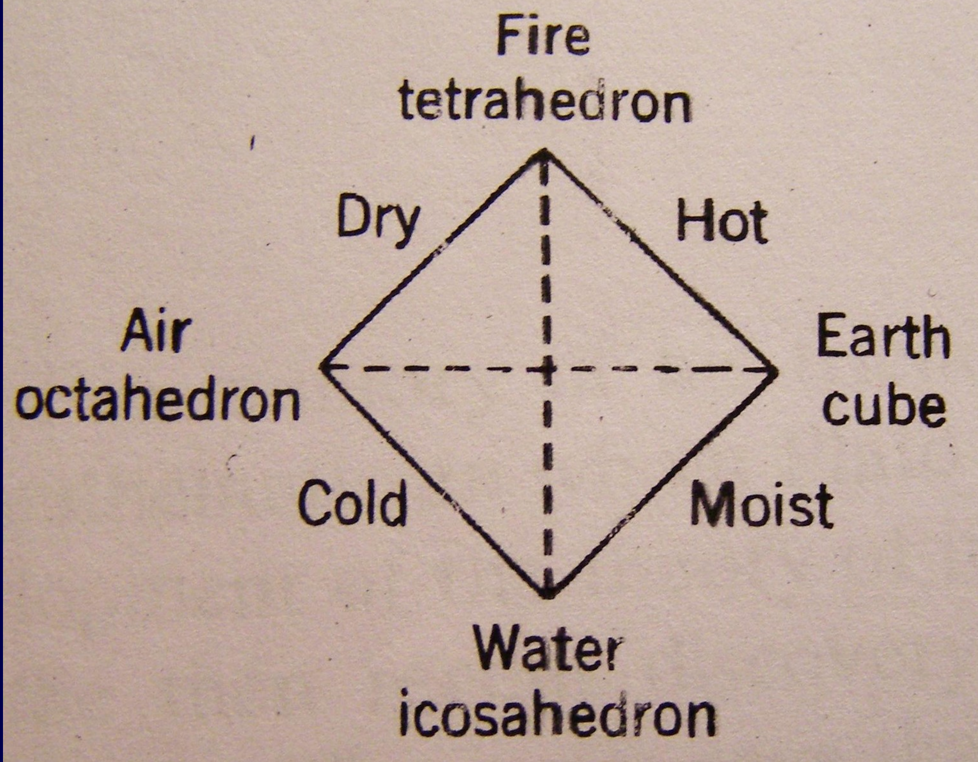
- Theaetetus: There are not more than five regular solids (proposition 18 book XIII)

Realization of the icosahedrons:

- Boron: B_{12} Molecule whose 12 atoms are arranged like the vertices of an icosahedron.
- Virus that causes de measles looks much like the icosahedron itself.

http://en.wikipedia.org/wiki/Boron
http://en.wikipedia.org/wiki/Measles

Plato's Cosmology

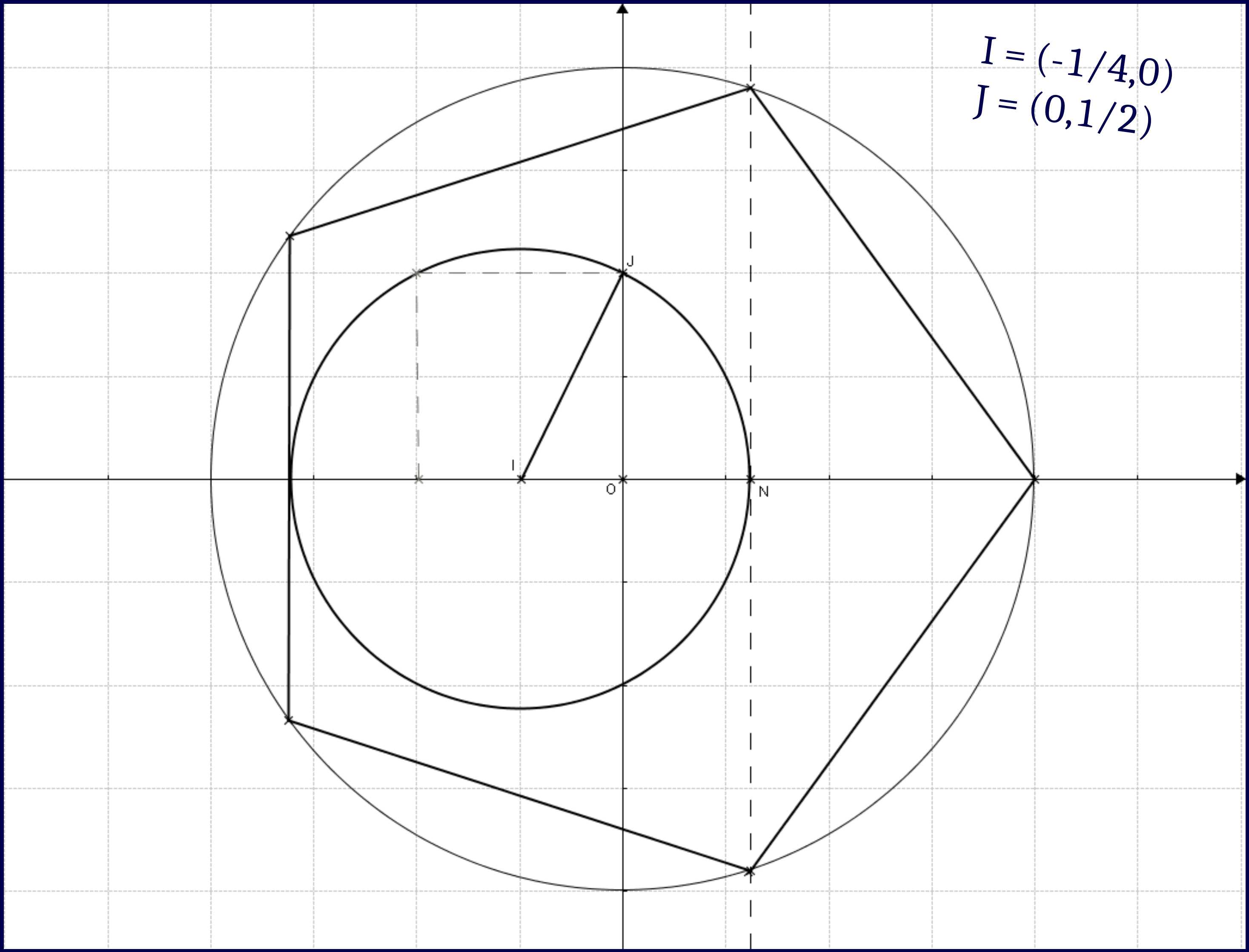


Elements and regular solids

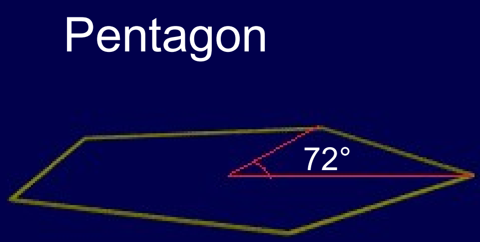
Construction:

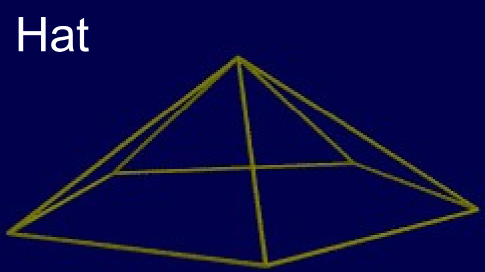
- *Proposition 16: To construct an icosahedron and comprehend it in a sphere, like the aforesaid figures; and to prove that the square on the side of the icosahedron is the irrational straight line called minor. (Euclid's Elements Book XIII)*
- http://aleph0.clarku.edu/~djoyce/java/elements/bookXIII/propXIII16.html

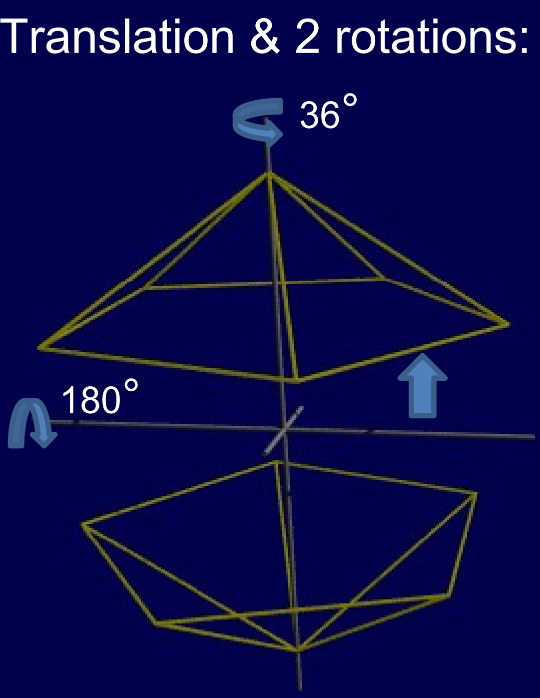
Ruler and Compass construction of a Pentagon:

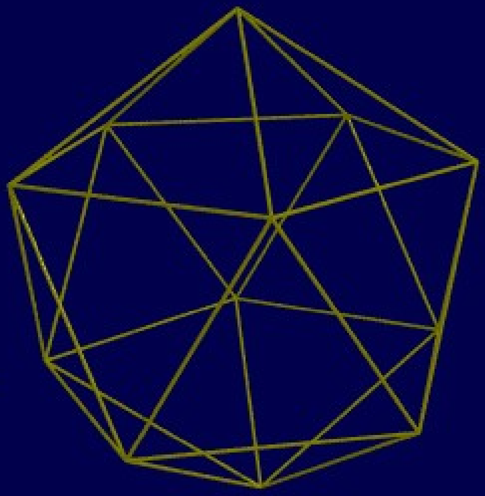


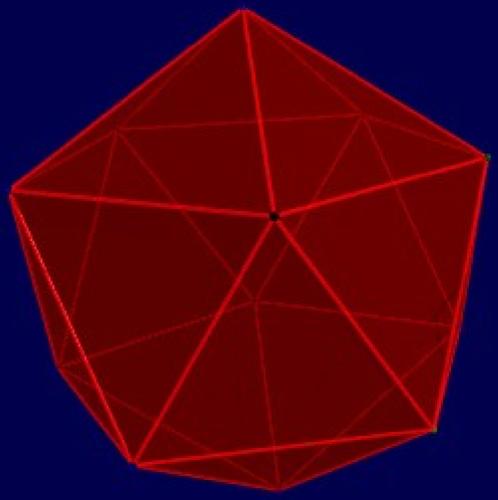
How to construct an Icosahedron:



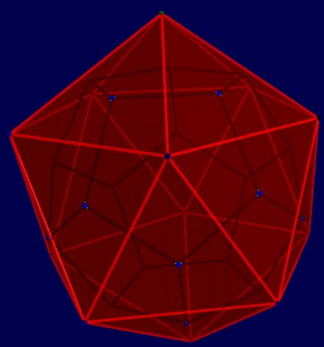






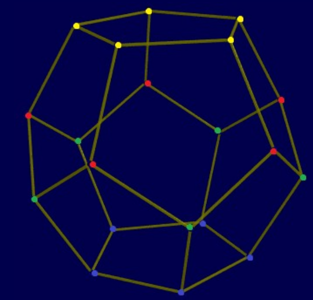


Universality of the Icosahedron:

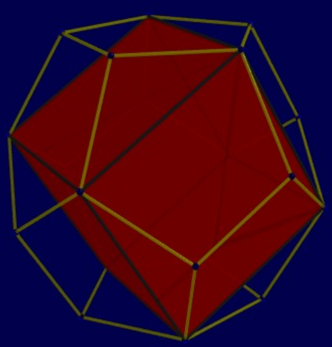


New vertices: center of gravity of the equilateral triangles

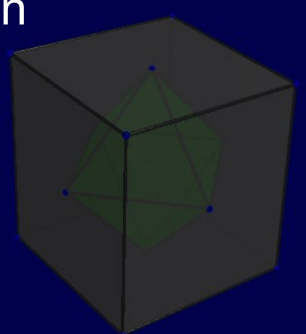
Dodecahedron



Cube

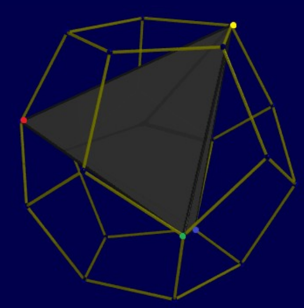


Octahedron

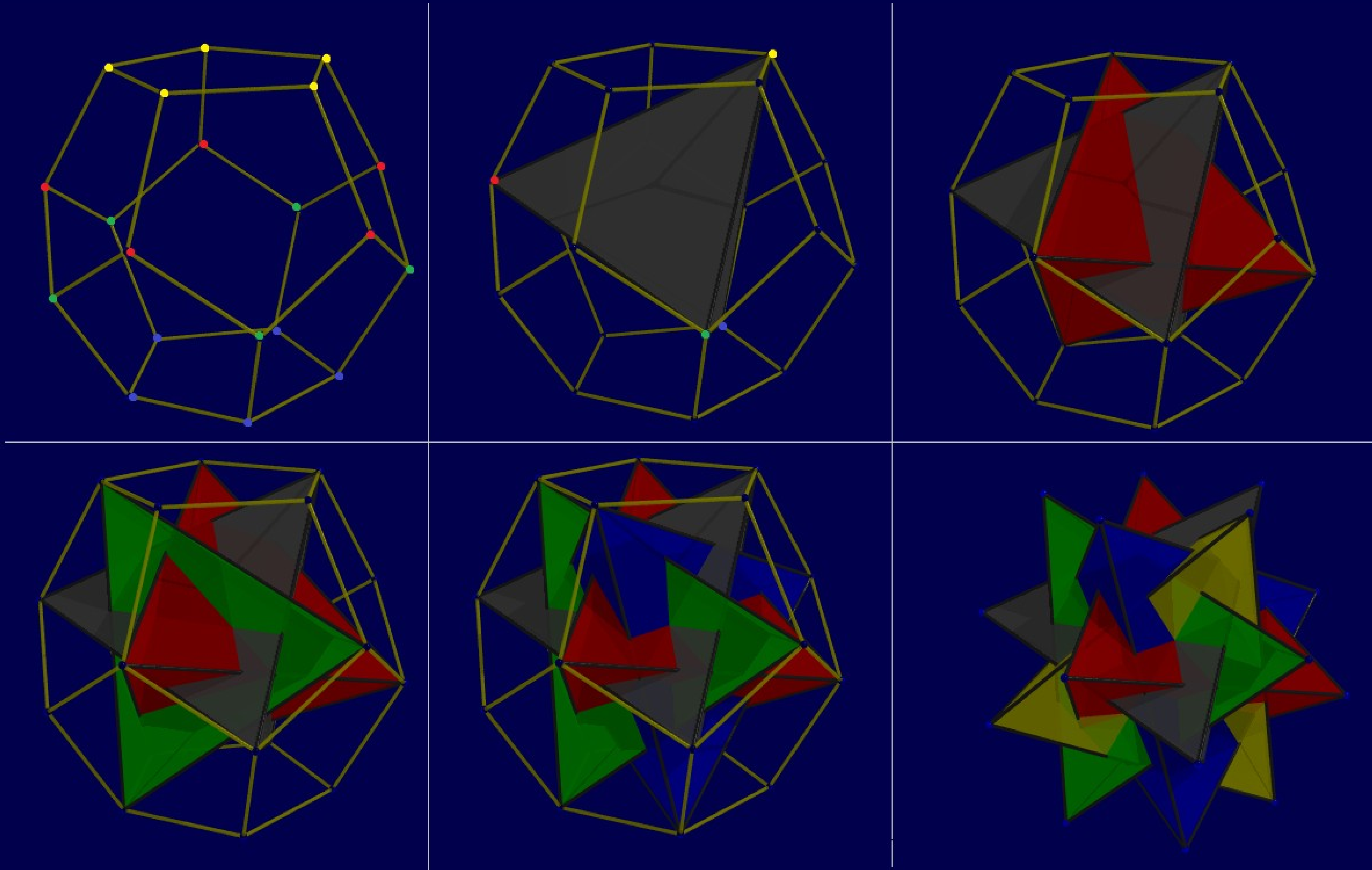


(New vertices: center of gravity of the squares)

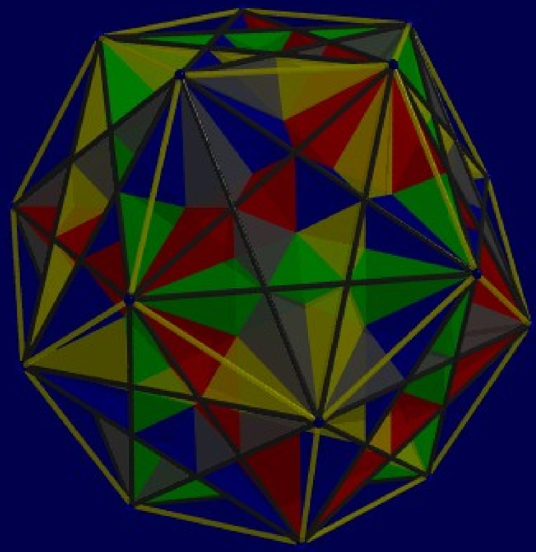
Tetrahedron

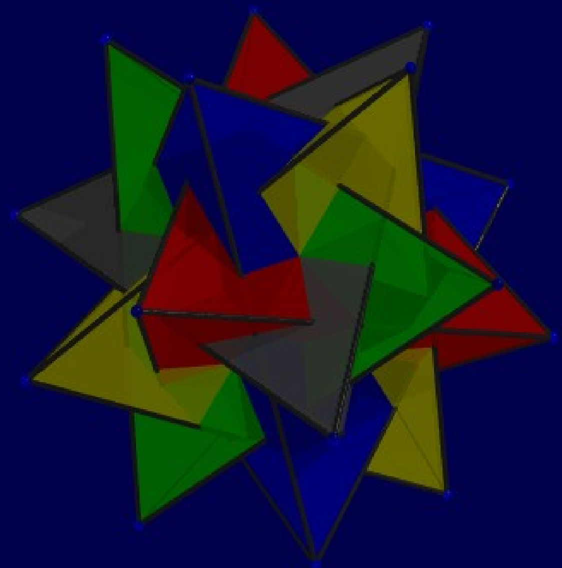


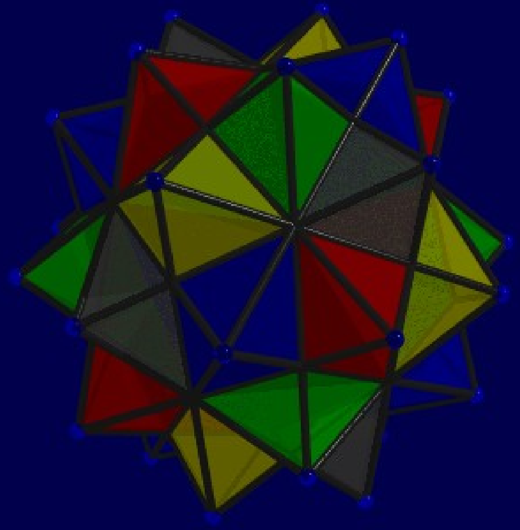
5 Tetrahedrons:



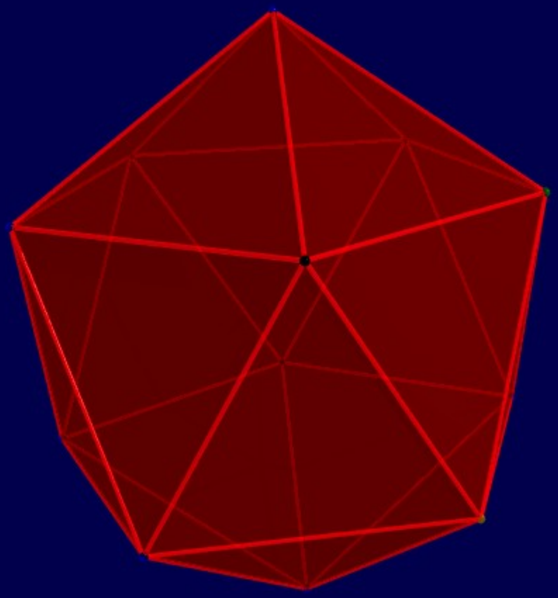
5 Solides:





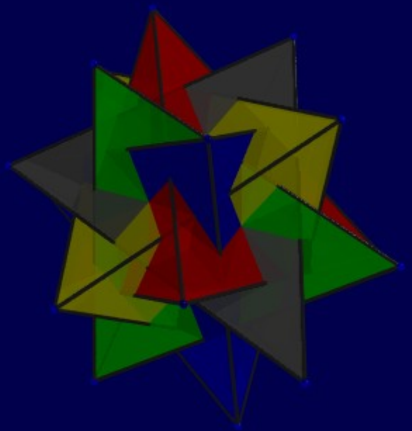


Icosahedral group:



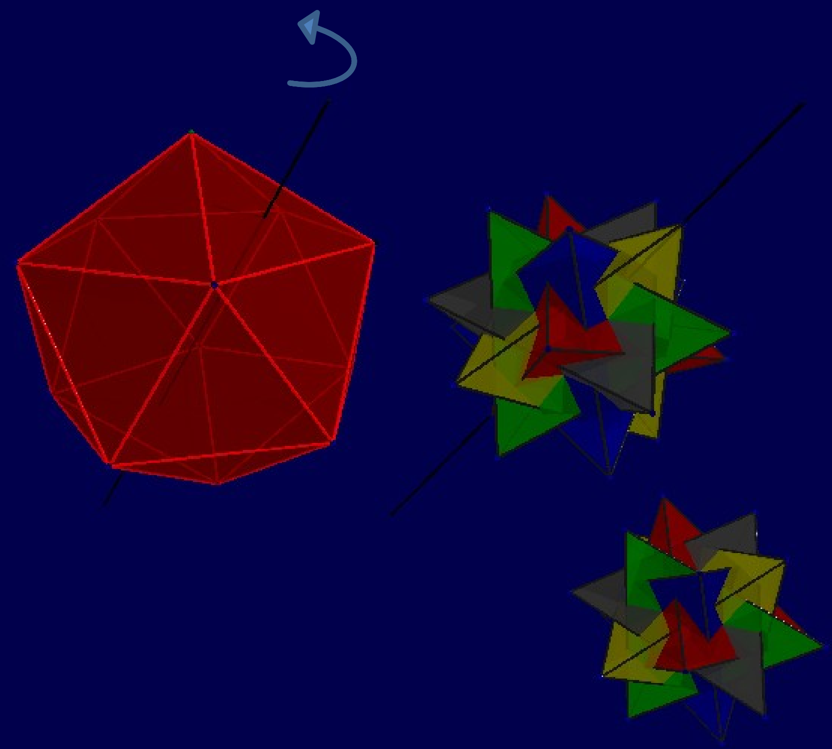
Isometries: distance preserving transformation of the Euclidean Space.

Permutations of the five colored tetrahedrons



An example:

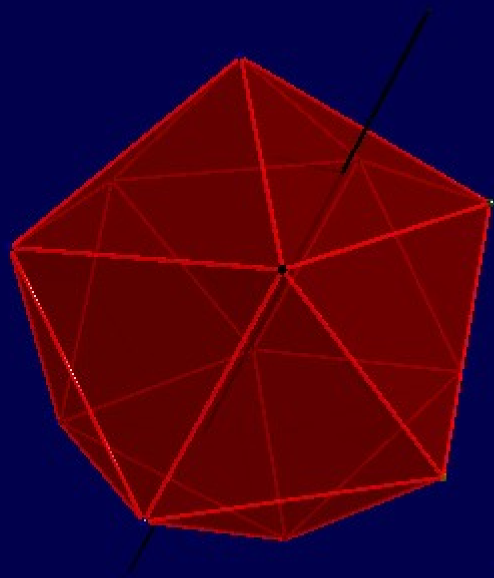
A rotation of 120° around an axe

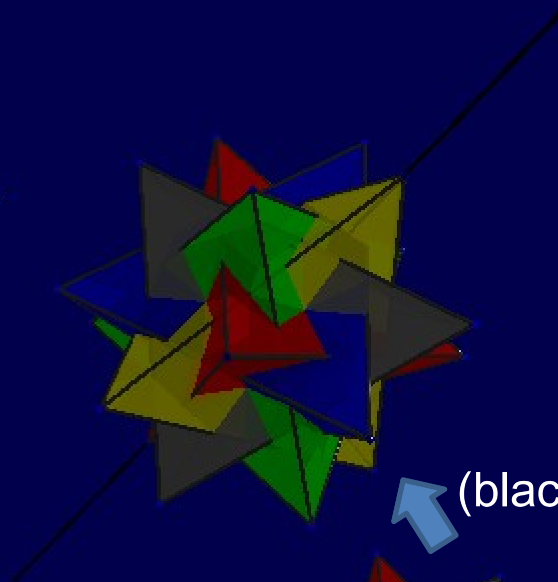


3-cycle:

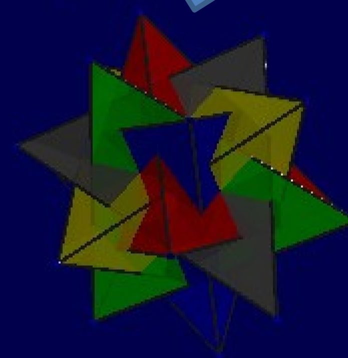
black	red	green	blue	yellow
blue	red	black	green	yellow

Representation of 3-cycles:

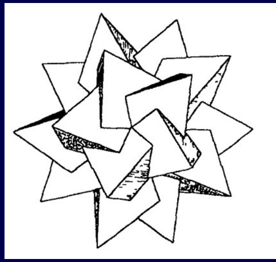




(black blue green)



- The axes pass through a vertex of the yellow and a vertex of the red tetrahedron
- Yellow & red are fixed by the permutation



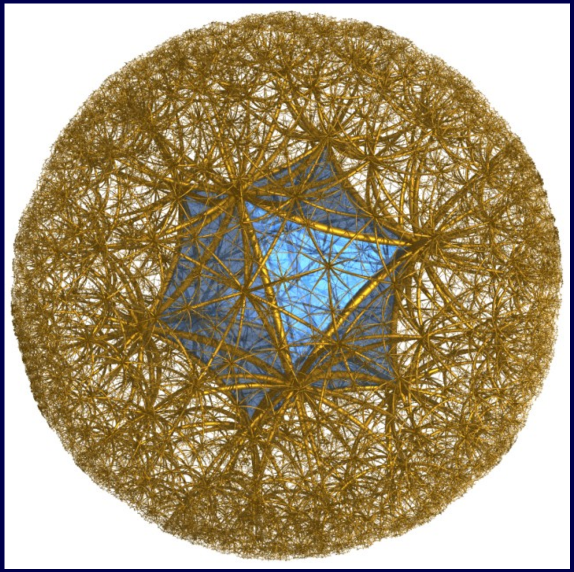
Group structure:

- 3-cycles generates the alternating group A_5
- All 3-cycle is represented by a rotation of 120° (positive isometry)

Proposition: The icosahedral group is isomorphic to the alternating group A_5

Poincaré's Theorem:

There is a compact 3-manifold having the homology groups S^3 but which is not simply connected.



Bredon Glen E. Topology and Geometry, Springer-Verlag, 1993

http://en.wikipedia.org/wiki/Icosahedron

Poincaré's conjecture:

Every simply connected closed 3-manifold without boundary is homeomorphic to a 3-sphere (S^3).

LADER Olivier

Pictures created with POV-Ray